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FIVE DIMENSIONAL COSMOLOGICAL MODEL WITH TIME-DEPENDENT G AND Λ

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$Abstract$

Five dimensional cosmological model is obtained in the presence of global equation of state of the form $p = \frac{1}{3}\rho\phi$ where ϕ is a scale factor, in the framework Kaluza -Klein theory of gravitation. An exact solution for matter distribution in cosmological models satisfying $G = G_0\left(\frac{a}{a_0}\right)^{n-2}$ is presented by using the method of solving the field equations is given by Ibotombi Singh at el. (2009). The corresponding physical interpretation of the cosmological solution are also discussed.

Keywords: Cosmology, higher dimensional space time, string cloud

1. Introduction

The study of higher dimensional cosmological model is motivated mainly by the possibility of geometrically unifying the fundamental interaction of the universe. In the context of Kaluza-Klein and super string theories higher dimension have acquired much significant. It has also been suggested that the experimental detection of the time variation of the fundamental constants could provide strong evidence for the existence of extra dimensions (Alvarez and Cavela , 1983; Marciano, 1984: Randjbar- Daemietal: 1984).

The added extra spatial and temporal type dimension are usual taken to be

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compact. A crucial point for this type of theory is to explain why the internal space spanned by extra coordinates is a so small that one cannot observed it. A possible solution to this is so called cosmological dimensional reduction, the contraction of the size of the supplementary coordinate to a Planck scale being a consequence of the dynamical evolution of the higher dimensional universe, as the result of the introduction of higher dimensional stress energy tensor.

The earlier suggestion of Kaluza-Klein regarding the topology of the extra dimension has now been replace by what is called 'spontaneous compactification' , where four- dimensional space time expands while the extra dimension contract to unobserved Planekian length scale or remain invariant (Chodos and Detweiler , 1980).

Several workers have obtained extra solution using higher dimensional space time for both cosmological and non-cosmological cases with or without matter (Chatterjee et al., 1990; Banerjee et al., 1990; Mayers and Perry, 1986).

A theory of gravitation using G and Λ as non constant coupling scalar has been used by Beesham (1986) and Abdel Rahman (1990). The motivation was to include a G coupling constant of gravity as a pioneer by Dirac in (1937).

In relativistic and the observational cosmology, the evaluation of the universe is described by Einstein's field equations together with perfect fluid and an equation of state. The Einstein's theory of gravity contains two parameters- Newtonian gravitational constant G and cosmological constant Λ. Normally, these are considered as fundamental constants. The gravitational constant G plays the role of a coupling constant between geometry of space and matter content in Einstein field equations. The Λ arises naturally in general-relativistic quantum field theory where it is interpreted as the energy density of the vacuum (Fulling et al., 1974).

If we assume the equality of gravitational and inertia mass and gravitational time dilation in Einstein theory we must required that the equation of motion of particle and photon does not contain G and Λ. In any case the strongest constraints are presently observed G_0 value and observational limit Λ_0. Sistero (1991) found an extra solution for zero pressure models satisfying $G = G_0 \left(\frac{a}{a_0}\right)^n$. Also Ibotombi and Anita (2007) investigated the Newtonian form gravity theories with varying G an exact solution and all asymptotic cosmological behavior is found for a universe with $G \propto t^{-n}$.

In the view of the above we consider in this paper the global equation of state $p = \frac{1}{3}\rho\phi$ where ϕ is a function of scale factor a. We present an exact solutions for the matter distribution in cosmological model satisfying $G = G_0 \left(\frac{a}{a_0}\right)^{n-2}$.

2. Model and Field Equation

We choose five dimensional spherically symetric metric of the form

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{(1-kr^2)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + (1-kr^2)\, d\varphi^2\right], \tag{1}$$

where k is the curvature index which can take up the values (-1, 0,+1) corresponding to the open, flat and closed universe respectively and $a(t)$ is the scale factor .
The energy momentum tensor T_{ij} in the presence of a perfect fluid has the form

$$T_{ij} = (p+\rho)u_i u_j + p g_{ij}, \tag{2}$$

where p and ρ are respectively, the energy and pressure of the cosmic fluid and u_i is the fluid five-velocity such that $u^i u_i = -1$.
Therefore, from equation (4) the energy momentum tensor T^i_j can be expressed as

$$T^o_o = -\rho, T^1_1 = T^2_2 = T^3_3 = T^4_4 = p. \tag{3}$$

For the perfect fluid distribution Einstein field equations with the cosmological constant Λ and gravitational constant G may be written as:

$$G^{ij} = R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G T^{ij} + \Lambda(t)g^{ij}. \tag{4}$$

Taking on the account the Bianchi identity $(G^{ij})_{;i} = 0$ and $(T^{ij})_{;i} = 0$ in the presence of cosmological constant term and gravitational constant gives from equation (4)

$$8\pi G_{;j}T^{ij} + \Lambda_{;j}g^{ij} = 0. \tag{5}$$

With help of equation (3), the Einstein field equations (4) for k = 0 can be expressed as

$$\frac{6\dot{a}^2}{a^2} = 8\pi\ G\rho + \Lambda, \tag{6}$$

$$\frac{3\ddot{a}}{a} + \frac{3\dot{a}^2}{a^2} = -8\pi Gp + \Lambda, \tag{7}$$

where dot indicates a derivative w. r. to. 't'.

With the help of equation (6) and equation (7) we get

$$4\frac{\dot{a}}{a}(p+\rho) + \dot{\rho} = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G}\right). \tag{8}$$

Equation (8) can be split to gives [Abdel Rahman (1990), Beesham (1986),]

$$4\frac{\dot{a}}{a}(p+\rho) + \dot{\rho} = 0, \tag{9}$$

$$\dot{\Lambda} + 8\pi\dot{G}\rho = 0. \tag{10}$$

Equation (9) can also be written as

$$\frac{d}{dt}\left(a^4\rho\right) + p\frac{d}{dt}\left(a^4\right) = 0. \tag{11}$$

We assume the global equation of state of the form

$$p = \frac{1}{3}\rho\phi, \tag{12}$$

where ϕ is a function of the scale factor $a(t)$.
From equation (11) and equation (12) we get

$$\frac{1}{\varphi}\frac{d\varphi}{dt} + \frac{4}{3}\frac{\phi}{a} = 0. \tag{13}$$

where

$$a^4\rho = \varphi. \tag{14}$$

If ϕ is a given explicit function of $a(t)$, then equation (12) is determined, and φ follows from equation (13)

$$\varphi = \varphi_0 \exp\left[-\int \frac{\phi}{a} da\right]. \tag{15}$$

Conversely, if φ is a given, then ϕ immediately follows from equation (13)

$$\phi = \frac{3}{4}\frac{a}{\varphi}\frac{d\varphi}{da}. \quad (16)$$

The Friedman equation (7) with the help of equation (14) gives,

$$6\dot{a}^2 = 8\pi G\varphi a^{-2} + \Lambda a^2. \quad (17)$$

Equation (10) and equation (13) with $\frac{d}{dt} = \dot{a}\left(\frac{d}{da}\right)$ gives

$$8\pi\frac{dG}{da} + \varphi^{-1}a^4\frac{d\Lambda}{da} = 0. \quad (18)$$

From equation (18) it is observed that if $G = G(a)$ is given then after integrating equation (18) we get value of $\Lambda = \Lambda(a)$. The equation (17) determines $a = a(t)$ and the problem is solved. Similarly if $\Lambda(a)$ may be given instead of $G(a)$ derives from equation (18) we get $G = G(a)$ in term of $a(t)$, after integrating of equation (17).
We solve equation (15) for a matter dominated universe and radiation dominated universe.

Case: I Matter Dominated Universe

If we consider $\phi = 0$, we get $p = 0$ then from equation (15) we get

$$\varphi = \varphi_0, \quad (19)$$

From equation (13) and (19), we have

$$\rho = \frac{\varphi_0}{a^4}. \quad (20)$$

Substitute the condition

$$G = G_0\left(\frac{a}{a_0}\right)^{n-2}, \quad (21)$$

into equation (18) with equation (19), we get

$$\Lambda = -\Lambda_0'\frac{(n-2)}{(n-6)}a^{(n-6)}, \quad (22)$$

where $\Lambda_0' = \frac{8\pi G_0\varphi_0'}{a_0^{n-2}}$.

After substituting the value of Λ from equation (22) in equation (17), we get

$$a(t) = \alpha_0 t^{\left(\frac{2}{6-n}\right)}, \tag{23}$$

where $\alpha_0 = \left[\frac{2}{6-n}\sqrt{\frac{\beta_0'}{6}}\right]^{\left(\frac{2}{6-n}\right)}$.

Equation (21) becomes

$$G = \frac{G_0}{a_0^{n-2}} \alpha_1 t^{\left(\frac{2(n-2)}{6-n}\right)}, \tag{24}$$

where $\alpha_1 = \left[\frac{2}{6-n}\sqrt{\frac{\beta_0'}{6}}\right]^{\left(\frac{2(n-2)}{6-n}\right)}$.

Using equation (12), (14) and equation (9), (10) yield

$$a(t) = \alpha_0 t^{\left(\frac{2}{6-n}\right)}. \tag{25}$$

$$\Lambda = -\Lambda_0' \frac{(n-2)}{(n-6)} \alpha_2 t^{(-2)}, \tag{26}$$

where $\alpha_2 = \left[\frac{2}{6-n}\sqrt{\frac{\beta_0'}{6}}\right]^{(-2)}$.

From equation (9) we get

$$\rho = \frac{k_2}{\left[\frac{2}{6-n}\sqrt{\frac{\beta_0'}{6}} t\right]^{\left(\frac{8}{6-n}\right)}}, \tag{27}$$

$$\rho(t) = F' t^{\left(\frac{-8}{6-n}\right)}, \tag{28}$$

where $F' = \frac{k_2}{\left[\frac{2}{6-n}\sqrt{\frac{\beta_0'}{6}}\right]^{\left(\frac{8}{6-n}\right)}}$.

Now from equation (10)

$$G(t) = \alpha_3 t^{\left(\frac{2n-4}{6-n}\right)}, \tag{29}$$

where $\alpha_3 = \frac{-B'(6-n)}{8\pi F'(n-2)}$.

Case: II Radiation Dominated Universe

If we consider $\phi = 1$, we get $p = \frac{1}{3}\rho\phi$, then from equation (15) we get

$$\varphi = \frac{\rho_0'}{a}, \tag{30}$$

where $\rho_0' = \rho_0 a_0^4$.
From equation (13) and (30), we have

$$\rho = \frac{\rho_0'}{a^5}. \tag{31}$$

Substitute the condition of equation (21) into equation (18) with equation (30), we get

$$\Lambda = -\Lambda_0 \frac{(n-2)}{(n-5)} a^{(n-7)}, \tag{32}$$

where $\Lambda_0 = \frac{8\pi G_0}{a_0^{n-2}} \rho_0'$.

After substituting the value of Λ from equation (32) in equation (17), we get

$$a(t) = \gamma_0 t^{\left(\frac{2}{7-n}\right)}, \tag{33}$$

where $\gamma_0 = \left[\frac{2}{7-n}\sqrt{\frac{\beta_0}{6}}\right]^{\left(\frac{2}{7-n}\right)}$.

Equation (21) becomes

$$G = \frac{G_0}{a_0^{n-2}} \gamma_1 t^{\left(\frac{2(n-2)}{7-n}\right)}, \tag{34}$$

where $\gamma_1 = \left[\frac{2}{7-n}\sqrt{\frac{\beta_0}{6}}\right]^{\left(\frac{2(n-2)}{7-n}\right)}$.

Using equation (12), (14) and equation (9), (10) yield

$$a(t) = \gamma_0 t^{\left(\frac{2}{7-n}\right)}. \tag{35}$$

$$\Lambda = -\Lambda_0' \frac{(n-2)}{(n-5)} \gamma_2 t^{(-2)}, \tag{36}$$

where $\gamma_2 = \left[\frac{2}{7-n}\sqrt{\frac{\beta_0'}{6}}\right]^{(-2)}$.

From equation (9) we get

$$\rho = \frac{k_1}{\left[\frac{2}{7-n}\sqrt{\frac{\beta_0}{6}}t\right]^{\left(\frac{8}{7-n}\right)}}, \tag{37}$$

$$\rho(t) = Ft^{\left(\frac{-8}{7-n}\right)}, \tag{38}$$

where $F = \frac{k_1}{\left[\frac{2}{7-n}\sqrt{\frac{\beta_0}{6}}\right]^{\left(\frac{8}{7-n}\right)}}$.

Now from equation (10)

$$G(t) = \gamma_3 t^{\left(\frac{2n-6}{7-n}\right)}, \tag{39}$$

where $\gamma_3 = \frac{-B(7-n)}{8\pi F(4n-11)}$.

3. CONCLUSION

We have presented here the solution of 5-D world in Kaluza-Klein theory with the global equation of state of the form $p = \frac{1}{3}(\rho\phi)$.
We find an exact solution of the matter distribution in cosmological model satisfying $G = G_0\left(\frac{a}{a_0}\right)^{n-2}$ by introducing the general method of solving the cosmological field equation. We discuss the two cases for $\phi = 0$ and $\phi = 1$ for dust and radiation model. For dust and radiation models, it is observed that cosmological constant $\Lambda \propto t^{-2}$ which matches with its natural dimension meaning their by no large a dimensional constant associated with the cosmological term in field equation. Cosmological constant Λ gradually increases as the universe expand. It is also observed that the gravitational constant G in either the cases can be decreases or increases function of t it depends on the value of n. In both cases energy density ρ is always positive. In $n = 2$ the cosmological constant vanishes in both the cases. The additional conservation equation (10) gives the coupling of the scalar field with matter.

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